

**COMPETITION COMPLEXITY AND STABILITY IN  
ORGANIZATIONS**

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## **INTRODUCTION**

In some senses, organizations are unambiguous: their objectives are to create payoffs, renew them in the face of rivalry from outside and entropy within, and to distribute them among the various players or stakeholders. If we agree on the definition of payoffs, success and failure can be distinguished succinctly: positive payoffs symbolize success, and zero payoffs, failure. Alternatively they are also open texts, presenting a field of possibilities, their aims and performance extending to a variety of interpretations, with none particularly dominant. Organization takes place at multiple overlapping levels; whole economies, firms, alliances, divisions, businesses, workplaces and so on. Apparent opposites coexist; equilibrium and disequilibrium, reversibility and irreversibility, stability and instability, areas of failure and success within the same entity.

The paper represents an attempt to capture the ambiguity of complex processes and provide a framework to describe the variables affecting payoffs. The aim is to present a framework which helps to understand a number of issues. How organizations grow and decline, and enter into, and out of periods of apparent stability and instability? Why, some times, is individual action of no consequence, and at others, critical? Another aim is to portray some of the complex interactions between strategic variables. Simple frameworks and models will probably always be adopted for practical purposes: but what are their limits? The relationship between partial frameworks and a more general archetype is discussed.

The expressions firm, organization, corporation are used interchangeably: neither boundaries between organizations, nor the frontiers between organizations and entire economies, national or global are completely clear. Given the network of alliances, joint ventures and mergers that evolve as a result of growth, decline, realignments and restructuring, the evolution and decay of new technologies and products, this is not surprising. The paper does not pretend to set out a positivist framework, with unbroken chains of cause and effect. Rather a method of approach is offered, that stresses interdependence and suggests that the scope and significance of individual action is both greater and less than many current approaches suggest.

### **Plan of the paper**

The paper develops in the following way. First, a general payoff function for organizations, and some complex interactions contained within it are described. The next section views organizations as a matrix of potential payoffs or synergies which corresponds, in some respects,

to a Boolean network. The third section examines the growth of payoffs over time. The last section summarizes the implications of the arguments in the paper.

## THE PAYOFF FUNCTION

In a general way an organization, or whole economies, can be thought of as a cooperative game with  $N$  agents or decision makers. Any subset,  $S$  or  $T \subseteq N$  is a coalition. Activities are primitives in the paper, and  $2^N$  coalitions between them are possible<sup>(1)</sup>. In the context of the entire economy coalitions can be looked upon as firms, and the growth, decline, extinction, and emergence of new businesses, can be looked upon as a process of forming and reorganizing coalitions. Alternatively, within organizations, sub coalitions of divisions, business units, reporting centres, and so on, can be built. For each coalition there is a single number, or payoff,  $Z(S)$ , or  $Z(T)$ , that is available to a group of agents, broadly including all stakeholders, workers, managers, customers, creditors, stockholders and the community. To simplify things, payoffs are assumed to be transferable: they can be measured in money, or in psychic terms (quality of life, security, a good environment) and distributed to stakeholders as transferable utilities. Payoffs measured in money terms are added values, corresponding to an economic surplus, or rent.

In any period the determinants of total payoffs,  $Z$ , viewed as potential rather than actual values (hence the inequality), can be expressed as

$$Z \# \{[P], [_{-s}], [_{-s}]_{\text{all } s \in N}; [A], [N], [C]; [H], [E], [_{-}]\} \quad (1).$$

The function can be divided into three sets of related variables:

- i. *decision variables*:  $_{-s}$ , and  $_{-s}$  are decisions and actions reflecting expected utilities in the preference relation  $[P]$ .  $_{-s}$  refers to decisions taken by  $S$ , an individual or group, and  $_{-s}$  those of others,  $-S$ . In turn, they determine *activity variables*, and their impact is conditioned by *environmental variables*.
- ii. *activity variables*:  $[A]$ ,  $[N]$ , and  $[C]$ , respectively, denote value adding activities, their number, and the coalitional structure of an entire economy or an organization.  $[A]$  is a matrix of activities which corresponds to the production functions or value chains of firms, that generate payoffs. The number of activities,  $[N]$ , describes the size, and scope of the economy, or organization. The way in which basic activities are grouped into firms or partitions within firms is described by  $[C]$ . Joint actions taken to by coalition members determine the size and division of payoffs.

iii. *environmental variables*: these are summarized by [E], [H], and [⊖]. [E] denotes expectations about the external environment, [H] is an irreversible state variable reflecting the history and path dependence of the system. [⊖] is a stochastic factor, incorporating shocks, or noise, emphasising the non deterministic nature of expression (1).

The payoff function can be described as an open text, in contrast to many of the partial and deterministic models developed in strategy, which focus on a restricted set of relationships, and assume an unbroken chain of cause and effect. The payoff function is seen as a network of inter relationships. Strategies are formed interactively at many levels; whole economies, organizations, divisions within organizations, business units within divisions, and so on. Organizations rise and fall, and are continually revised and recombined as a result of competition, integration, and coordination. Learning and adaptation results in the emergence of new industries and products, and the development of new niches. The system encompassed by the payoff function is an example of a non linear adaptive network, with interactions between many changing agents, bounded rationality, and positive feedbacks in the form of increasing returns and irreversibilities.

Strategic models and frameworks whilst recognizing interactions, naturally partition variables into groups akin to Figure 1(a). Choice models, focus on decision variables; the traditional approach stressing planning and intentional, sequential processes; or fashioning strategy gradually, by trial and error, the behavioral approach. Other models emphasise the importance of activity variables; low cost and differentiation, integration and responsiveness, mergers and alliances. In others environmental variables, like industry conditions, rivalry, and macroeconomic factors have precedence. Interdependence beyond single triangles in Figure 1(a) is acknowledged, but in the interests of simplicity, its importance is minimised. One reason for the transience of fads and fashions in strategy, is that excluded connections are either significant, or become so, and start to overwhelm prescriptions and predictions based on simpler connections<sup>(2)</sup>.

FIGURE 1: Patterns of interdependence

Kaufman's image of *coadaptation on a dancing landscape* can be reinterpreted in terms of the payoff function. Changes in variables in one area alter conditions in the surrounding niche, and perhaps in the entire landscape: the greater the connectivity between elements, the more complex the system and the greater the potential, either for decline, as each species or

organization is cast back to some average state (or fitness), or evolution, as new opportunities are created. Two further observations are suggested by Kaufman's image. The first is that the payoff function may have varying sensitivity to changes in variables: in some phases it may be more temperamental than others. This possibility is examined later in the paper.

### **Standard operating procedures**

The second observation is that agents, in the adaptive non linear networks being described, do not act simply in terms of stimulus and response. They anticipate, and adopt procedures which prescribe actions under specific conditions. In this context, Holland speaks of standard operating procedures.

Broadly the standard procedures adopted by organizations can be classified in terms of governance mechanisms. Ghoshal and Nohria (1993) describe three such mechanisms. *Centralization*, which refers to the role of formal authority and hierarchies, is the first. The second is *formalization*, which includes decision making through bureaucracy, rule books, prescribed processes. *Normative integration* is the third. This relies on socialization, "*shared goals, values and beliefs that shape.... [decision makers]...perspectives and beliefs*". Given multiple decision makers, acting on a large number of variables, the potential for disorder is immense. The three procedures describe different ways of meeting unforeseen circumstances, and try to stipulate, ex ante, how an organization will react when they arise. In general the three mechanisms can be seen as adaptations, which serve the purpose of reducing the dimensionality of the payoff function, by confining the choices of multiple decision makers within a set of predictable routines and behaviours. They may result in attractors in the form equilibrium payoffs, periodic and quasi periodic orbits, or strange attractors, which are sensitive to initial conditions.

The role of governance mechanisms is important for two reasons. The first relates to the dimensionality of the function (1): the second to the realization of payoffs. Firstly, they may reduce the dimensionality of the payoff function at any level of generality, and even a chaotic system, of low dimension is more orderly and tractable than one that wanders through vast tracts of space on high dimensional attractors. Secondly, payoffs described in (1) are potential, and one role of organizations is to realize them through cooperation and trust.

## Norms, culture and intent

The third mode of governance, *normative integration*, is interesting since it describes ways in which the dimensionality of the decision making set, [P],  $\_s$ , and  $\_s$  is reduced. To developing the ideas of Nozick (1995), consider three types of utility; i. *instrumental*, ii. *conditional*, and iii. *symbolic*. An underlying assumption for each specification is bounded rationality, (Simon, 1982; 1990), which takes into account the cognitive limitations of both knowledge, and computational capacity.

*Instrumental utilities* relate strategic actions directly to expected payoffs, and are the product of the utility associated with the outcome, and the subjective probability of its occurrence. If  $\_s$  is an action taken by agent S which has an expected outcome  $O_i$ , expected instrumental utility is,

*Definition (expected instrumental utility):*

$$EIU(\_s) = \sum_{i \in N} \text{prob } U(O_i) \quad (2).$$

Expected payoffs are not influenced by the choice of action itself. Hence the utility, expected by decision maker S, depends only upon the subjective probability S attaches to a particular outcome, which is independent of any action that another agent, his rival or partner, -S, might take.

*Conditional utilities* indicate one individual's perceptions of another's motivation, and the supposition that, faced with similar alternatives, others will respond in the same way as herself. Expected conditional payoffs are the product of the utility associated with a particular outcome, and the subjective conditional probability of that outcome occurring. Hence the conditional probability which S attaches to a cooperative outcome is determined her own choices. Furthermore S can increase the probability of cooperative behaviour on the part of others by demonstrating cooperative behaviour herself. Expected conditional utility,  $ECU(\_s)$ , is,

*Definition (expected conditional utility):*  $ECU(\_s) = \sum_{i \in N} \text{prob } (O_i^* \_s) U(O_i)$

(3).

*Symbolic utilities* arise when a probable outcome of an action is associated with another outcome to which the individual attaches utility. Using the previous example, S may cooperate for a number of reasons; perhaps he attaches utility to having a reputation for being a

trustworthy partner, making a prestigious product, or being associated with a charismatic leader. Expected symbolic utility,  $ERU(\_s)$ , of action  $\_$  taken by S is,

$$\text{Definition (expected symbolic utility): } ERU(\_s) = \sum_{(i \in N)} \text{prob}(O_i) U(O_i + R) \quad (4).$$

where R is the event that agent S associates with outcome  $O_i$ .

Expected instrumental utilities are associated with individual maximising behaviour, and attractors, taking the form of Nash equilibria. Where there are multiple equilibria, decisions may cluster around focal points<sup>(3)</sup> which are conditioned by shared experiences and culture. Conditional utilities can be linked to culture, since this can be defined in terms of high probabilities of certain patterns of behaviour culture. In cultures based on trust, agents have formed the expectation that cooperative actions will be reciprocated and rewarded (and evidence of non cooperative actions punished). Symbolic utility is often summarized in mission statements, and enunciations of strategic intent, that link corporate goals to personal objectives valued by individual stakeholders.

Having outlined the payoff function, the next section examines payoffs as a matrix at a point in time. Later, possible trajectories are examined.

## ORGANIZATIONS AS LANDSCAPES

Consider an economy or organisation as being made up of a set of N activities. Value is added in two ways; (i) by stand alone entities,  $a_{ij}$ , ( $i=j$ ), and (ii) by a set of potentially synergistic relationships,  $a_{ij}$  ( $i \neq j$ ). The problem of interdependence is simplified by treating synergies as separate products (or services). The value of the whole,  $Z(W)$ , becomes the sum of these contributions:

$$Z(W) = \sum_{(i,j \in N)} a_{ij} \quad (5)$$

Variables  $a_{ij}$  represent the value added, or more precisely the (economic) rentals of activities, a surplus of returns over cost. A snapshot is an apposite description of inequality (5): coalitional structure is in the foreground, against the backdrop of other strategic variables in the payoff function (1). The matrix summarizes the *state* of the organization, as determined by strategic variables and decisions of the past: it is the imprint of history<sup>(4)</sup>. Normalized around rent no rent situations (1,0), activity matrices are akin to Boolean networks.

The significance of treating synergies as joint outputs should also be noted. Suppose every activity or coalition of activities is increased by a scalar,  $\alpha > 0$ , then its rental is increased by  $\alpha Z(W)$ : there are constant returns to each coalition (*linearity*). Equally, when activities, or coalitions are linked, there may be increasing or diminishing returns to cooperation. Following Arrow's treatment of externalities (1969), the organization matrix encompasses a subtle distinction, between constant returns (additivity and divisibility of payoffs) to particular activities or coalitions, and the possibility of increasing returns, or superadditivity (if synergies are positive), when they are combined.

In Figure 2 diagonal elements are stand alone (security) values. Off diagonals depicting synergies, may be positive or negative (or zero). Figure 2(a) can illustrate a whole economy or organizations within it. Payoffs, in terms of rent, grow and decline, alternating, for particular organizations, between positive and zero, as economies evolve through a process of creative destruction. Growth in overall payoffs is consistent, and perhaps contingent upon, decline in part of the matrix, as industries firms, institutions, and activities prosper, mature, and die, and are replaced.

This section is concerned with the process of coalition formation given a static set of payoffs, and much of the illustration is in terms of the (superficially) trivial matrices in 2(b) and 2(c). Payoffs to the grand coalition in 2(b) are 10, ( $z\{1,2\} = 10$ ). Payoffs to single member coalitions, ( $z\{1\}=2, z\{2\}=2$ ), are *security levels*, assumed to be the lowest that can be forced on individual agents:  $z(i) = 0$ .

FIGURE 2: Activity matrices

Interdependence is two way: one activity potentially creates value in others: similarly, it permits others to create value. This might be described as the yin and yang of synergy. Neither would be possible if the organisation were *unbundled*, or partitioned in such a way that businesses were effectively isolated. In the matrix, synergies are off diagonal elements ( $s_i, s_j$ ): inward synergies,  $a_{is}$ , are the contributions to value added in business  $s$  by other activities in the network, and outward synergies,  $a_{sj}$ , the contribution of  $s$  to others. Total synergies in  $s$  are the sum of the two:

$$\alpha^s = a_{is} + a_{sj} \quad (6).$$

The marginal contribution of an activity to a grand coalition is its value as a stand alone unit,  $a_{ss}$ , plus the value of inward and outward synergies:

$$m(a^s) = a_{is} + a_{sj} + a_{ss} \quad (7).$$

In economic terms the marginal value of an activity is its contribution to economic rent of a particular coalition; the difference to the rental of a coalition if it were withdrawn. In Figure 2(b), the marginal contribution of each activity as a one agent coalition is  $z\{1\} - z\{1\} = 2$ , and  $z\{2\} - z\{1\} = 8$ , and a two agent coalition marginal contribution are,  $z\{1,2\} - z\{1\} = 8$ ,  $z\{1,2\} - z\{2\} = 8$ : reciprocal synergies created by each activity are 4. In Figure 2(c), synergies created by the three activities to the matrix as a whole (the grand coalition), are (22, 20, 18) respectively, and the marginal value of each activity to the grand coalition is (26, 25, 24).

### **Improvements and efficiency**

In Kaufman's metaphor of the development of a technology on a rugged fitness landscape, early on, when radical improvements are easier, a series of long jumps take place. Then major change becomes more difficult and local hill climbs begin, bringing incremental improvements, but waiting times increase between further long jumps to more distant optima. The activity matrix is broad enough to illustrate improvements in the form of increases in economic rent, in entire economies, organizations or partitions (divisions, business units, workplaces) and encompass all value adding activities<sup>(5)</sup>.

The metaphor of coalition formation cooperative games is useful in describing the principles behind improving value added. In the static framework there are three levels at which improvements in the form of increased rents can happen: i. viability; ii. convexity; iii. efficiency.

Given the assumption that no activity can be coerced into accepting less than its stand alone value, the weakest condition for improvement satisfying is viability. At any level of organisation is viable if two conditions are met: i. positive added value; and ii. it must be worth more than the sum of the constituent activities.

The next level of improvement is convexity. To guarantee an improvement a coalition should not accept activities unless their marginal value to the coalition is positive. Activity matrices which satisfy the condition that the bigger the coalition joined by an activity, the bigger the

contribution, are described as convex, in sense that there are increasing returns to cooperation. Convexity means that for any activity  $s$ , and any two coalitions,  $S$  and  $T \subseteq N \setminus \{s\}$ ,

$$z(S \cup T) \geq z(S) + z(T) - z(S \cap T) \quad (8)$$

At any level of organization a structure is efficient in a static sense (for given values of the variables in the payoff function) if, it is feasible, and if value added or rent cannot be merely by forming new coalitions. So defined, an efficient structure can be seen as an application of the idea of the core of a cooperative game to organizations. If  $x$  is an allocation to an activity the core of an organization  $S$  is defined as

$$\sum_{i \in S} x_i = z(S) \text{ and } \sum_{i \in S} x_i \geq z(S) \text{ all } S \subseteq N.$$

The condition for efficiency of organizations as mapped out in expression (5) is that they should be convex structures(7): this holds at any level of generality(7).

The core of the organization matrix in Figure 2(b) is illustrated in the simplex in Figure 3. It lies in the shaded area ABCDEF. The variables in 2(b) have been redefined: imputed rent to any activity must be no less than its the stand alone value. So  $Z(S^*) = z(S)$ : only synergies are considered the inequalities must hold:  $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$ .

Vertices in Figure 3 denote the sum of the off diagonal elements, the total synergy available to the grand coalition. Rent attributed in the grand coalition,  $Z^*(N) = x_1 + x_2 + x_3 = 30$ , must be at least as great as that to any proper coalition ( $S \subseteq N$ ):

$$x_1 + x_2 \geq 12, x_1 + x_3 \geq 10, x_2 + x_3 \geq 8,$$

### The random Invisible Hand

Using the Shapley value (Shapley, 1953) a unique value can be assigned to each activity in organizations at any level.

Definition : the Shapley value,  $v_k$ . This allocates value added by the grand coalition  $Z(N)$  among all  $k$  businesses as follows:

$$v_k = \sum_{(S \subseteq N, k \in S)} \frac{s!(N-s-1)!}{(N!)^2} [Z(S \cup \{k\}) - Z(S)]$$

$$\text{with } 0! = 1, Z(\emptyset) = 0, \text{ and } \sum_{k \in N} v_k = Z(N) \quad (9).$$

Expression (9) is the expected marginal contribution of an activity,  $b^k$ , to a coalition  $S$  consisting of  $s$  activities ( $*S*=s$ ). The right hand term is the marginal contribution of the activity to the appropriate coalition. On the left is the probability of the coalition occurring randomly,  $\frac{s!(N-s-1)!}{N!}$ . A random coalition is got by figuratively arranging all  $N$  businesses in a line: there are  $N!$  such orderings, each assumed to be equally likely. The probability weighting is the number of ways that coalition  $S$  can precede  $a^k$  in the ordering as a proportion of the total number of orderings<sup>(4)</sup>.

Following Shapley, there is exactly one function, corresponding to the properties of the organisation matrix which attributes a unique payoff to each player or business in the corporate structure game: this is the Shapley value, associating each business,  $k$ , given the underlying organisational matrix, with a unique payoff. The sum of these unique payoffs, is the value of the organization matrix  $Z(B)$ . It provides a means of (i). attributing value to businesses in the presence of synergies, and (ii). rewarding them in a way which corresponds to their power to create synergies<sup>(8)</sup>. The Shapley value of activity 1,  $v_1$  in the three activity game in Figure 2(c) is calculated as follows:

$$v_1 = 1/3z(1) + 1/6[z(1,2) - z(2) + z(1,3) - z(3)] + 1/3[(Z(N) - z(2,3))]$$

$$v_1 = 1/3(4) + 1/6[21 - 5 + 20 - 1] + 1/3[45 - 19] = 15.$$

Similarly  $v_2 = 15$  and  $v_3 = 15$ , giving  $\sum_{k=1,2,3} v_k = Z(N) = 45$ .

The Shapley value is located at  $V$  in Figure 3. The extraordinary result is provided by the Shapley value is that if coalitions are formed randomly according to the process described above, in a static system of payoffs, the expected value of payoffs to each activity sums to the value of the organization matrix.

FIGURE 3 The core and Shapley value

## TRAJECTORIES

A static framework was presented in the last section. Now consider developments over time. Potential complexity inherent in (1) is illustrated by using a first order difference equation

$$Z_{t+1} = f(Z_t) \tag{9}$$

Imagine two assumptions. Firstly, suppose there is some maximum rental  $Z$  that can be achieved by an organization. A limit for payoffs may exist for any number of reasons; competitive pressure may be an increasing function of payoffs for individual organizations or

for the system as a whole; or we might imagine a Richardian process in which investment opportunities decline as the system expands towards greater and greater profits bringing the cycle of accumulation to an end. Secondly, suppose that the growth of payoffs depends on two parameters,  $b$  and  $c$ , and how close actual payoffs,  $Z_t$ , are to maximum payoffs,  $Z$ . The growth of payoffs  $Z_t$  to  $Z_{t+1}$  is

$$Z_{t+1} = (1+b)[c(Z - Z_t)Z_t].$$

Denoting relative payoffs as  $\underline{z}_t = Z_t/Z$ , and an effective growth rate  $k = (1+b)CZ$ , equation (2) can be written as the logistic equation,

$$\underline{z}_{t+1} = g(\underline{z}_t) / k_{\underline{z}_t} (1 - \underline{z}_t) \quad (10).$$

Although (10) expresses quadratic behaviour, the simplest kind of non linearity, the solution is complex and dependent upon values of the parameter  $k$ . Relative payoff  $\underline{z}_t$  is a number  $0 < \underline{z}_t \leq 1$ , so the function  $g$  can never exceed a value of unity. Since the maximum value of  $g$  is  $g(1/2) = 1/4k$ , we must assume that  $0 \leq k \leq 4$ . The value of  $k$  depends upon i. the variables listed in the payoff function (1) and ii. interaction between them.

If  $0 \leq k \leq 3$  the system is similar to that of a linear difference equation. There are two fixed points in the process described by (10),  $x_1^* = 0$ , and  $x_2^* = (k - 1)/k$ . For  $k < 1$  the origin is stable. In Figure 4(a)  $k < 1$ . If we start at  $\underline{z}_0$ , the height  $g(\underline{z}_0) = \underline{z}_1$ , gives the second state of the system, and the corresponding point,  $\underline{z}_1$ , is reflected onto the horizontal axis by the  $45^\circ$  line. Further points in the evolution of the system can be found by projecting the next state  $g(\underline{z}_1)$  on to the  $45^\circ$  line. Successive points in the path get closer and closer to the origin.

FIGURE 4: Fixed points

At  $k = 1$  there is a change in stability,  $x_1^*$  becomes unstable. For  $1 \leq k \leq 3$ ,  $x_2^*$  is now stable<sup>(8)</sup>. This case is illustrated by Figure 4(b). When  $k > 1$  the height of the curve increases and the graph of  $g(\underline{z}_t)$  crosses the  $45^\circ$  at two points, the origin and  $g(x_2^*) = (k - 1)/k$  and the system converges to  $x_2^*$ .

At  $k = 3$  the two fixed points become unstable. A stable 2 cycle is born and payoffs alternate between high and low values. This period doubling bifurcation is interesting, but pales in comparison to the period doubling cascade which follows. At  $k = 1 + \sqrt{6}$  the initially stable two

cycle loses stability and a new 4 period cycle is created, following the same pattern. The system goes through an infinite number of such period doublings. Cycles of length, 8, 14, 32, ...,  $2^n$ , ... appear successively and become unstable. This all happens before a limit point  $k_4$  that is approximately equal to 3.57.

Between  $k = 3$  and  $k = 4$  cycles of all lengths appear, and the 3 cycle is fairly typical. As  $k$  approaches  $k_4$ , the limit point of the 2 period cycles, intermittent chaos sets in. The system alternates between an approximate 3 cycle, made up of an infinite number of period doublings, 6, 12, 24, ...,  $2^n \times 3$ ..., interspersed with chaotic behaviour. The set of  $k$  values which begin the 3 cycle up to the limit point resembling  $k_4$  of the series of bifurcations are termed period doubling windows,  $\omega_i$ , which are illustrated in Figure 5(a). Inside the windows, motion is asymptotic; outside there are no stable period orbits but an infinity of unstable ones. The logistic equation illustrates different kinds of attractors; equilibrium points,  $x_1^*$  and  $x_2^*$ ; periodic cycles of 2, 3, and so on; and strange attractors, orbits that are sensitive to initial conditions, in the sense that although they originate from points arbitrarily near one another, they become exponentially separated over time. Two kinds of sensitive dependence characterise chaotic systems: i. *sensitive dependence upon initial conditions* and ii. *sensitive dependence on the parameter  $k$*

At  $k = 4$ , for example, motion becomes chaotic, and has the property of *sensitive dependence upon initial conditions (SDIC)*. Practical attempts to observe the system will involve errors in initial measurement and rounding errors. Because of the sensitive dependence on initial conditions, such errors will be important and will introduce significant and unpredictable errors into forecasts made about the system. In the chaotic interval  $k_4 < k < 4$ , there is *sensitive dependence on the parameter  $k$  (SDP)*: arbitrarily small changes in  $k$ , brought about by changes in the payoff function lead to drastic changes in the behaviour of the system.

FIGURE 5: Bifurcations and so on

## Sensitive dependence on the parameter k (SDP)

Summarizing, we have four alternative stages which depend on the size of the parameter k:

- i. a stable phase  $0 < k < 1$  ending in zero payoffs.
- ii. in the region  $1 < k < 3$  stable payoffs from one period to the next with  $\lambda^* = (k - 1)/k$ .
- iii.  $3 < k < k_4$ , a phase of periodic attractors, interspersed with instability, as the system moves to successively higher 2 cycles.
- iv. in the interval  $k_4 < k < 4$ , the system goes through cycles of all phases (beginning 3), in an infinite number of small windows. Outside the windows there are no stable periodic orbits but an infinite number of unstable ones. At  $k = 4$  the system culminates in sensitive dependence upon initial conditions.

Stability is the characteristic of the first two stages. The second pair are unstable, but more important than the sensitivity to initial conditions at  $k = 4$  is sensitive dependence on the parameter k itself. The implications of SDP may be more serious than SDIC since a statistical averaging process can be used when chaos and SDIC exist in a deterministic system. Variations in the parameter k can make such averages unstable however, since they may be very different under due to the volatility of k even though they are close in payoff (Z and  $\lambda$ ) space.

The ambiguity referred to in the introduction is captured by the payoff function (1) in conjunction with a deterministic non linear dynamic of which () is just a simple example. Using () as an illustration variations in k, the effective growth rate, unify apparent contradictions; equilibria and disequilibria, stability and instability, success and failure at different levels of organization, order and disorder.

## CONCLUDING REMARKS

Since it is a composite variable in that it is determined by a number of variables, changes in k are caused by i. variations in the elements of the payoff function and ii. correlations between them. the greater the correlation between variables. Generally high values of k ( $3 < k < k_4$ , especially  $k_4 < k < 4$ ) and high correlations mean signify complexity, and sensitivity to changes in k itself.

- a. At any level of organization, relative stability is associated with low rates of effective growth and independence between variables in the payoff function: however lower levels are unlikely to experience calm given turbulence at higher levels.
- b. High interdependence is associated with greater potential turbulence.
- c. Low levels of effective growth may be unstable in the sense that individual organizations are caught in a trap that ends in zero payoffs and replacement by new organizations. The higher the level of organization of this low level equilibrium trap the more traumatic the effect of its death.
- d. The role of standard operating procedures, is to lesson the impact of interdependence by coordinating responses: markets are mechanisms for achieving this, but often high powered market like incentives are destructive of cooperation and learning.
- e. Managing at the edge of chaos has become a cliché. The idea that organizations "*must be driven*" far from equilibrium is an over deterministic reading of the open text provided by the metaphor of complexity. Values of the variables determining  $k$  are a matter of personal choice. The complexity of the payoff function are beyond the scope of individual choice. Reading the open text suggests that processes are too mysterious to be driven.
- f. Whether a regime or an organization is unstable, through SDP or SDIC, is something of an unforeseeable event. A single word or an act may, or may not be significant; topple one regime, create another. The importance of the individual is reinstated by this uncertainty.

#### NOTES

(1) Activities are the building blocks of organisations. Activity is a general category. It may refer to an operating company, a retail store; a reporting or profit centre, a functional area, or a strategic business unit that co-ordinates and controls production in a divisionalised corporation or more basically a sub component of the value chain. It corresponds to a discrete set of value chains, which can be jointly managed, on a more or less independent basis, and linked horizontally, and vertically, to other parts of the organisation: process, defined as "*a collection of linked activities that take one or more input and creates an output of value to the customer*" (Hammer and Champy, 1993), is a close approximation.

(2) It is by no means clear which of the networks in Figure 1 is the most complex. See Gell-Mann (1994, pp. 30-33).

(3) Dealing with multiple potential Nash equilibria, Schelling, examined conditions that may cause players in a game to expect each other to implement a particular equilibrium. When this happens, the expected equilibrium becomes self fulfilling. Shelling called this phenomenon a focal point effect. A focal point equilibrium has some property that conspicuously distinguishes it from all others. Many examples of focal points are provided in Schellings book. Simple qualitative factors underlie focal points. *Symmetry* and *uniqueness* seem to be important in selecting principles. In games involving division of payoffs, the symmetric notions of *equity*, or *efficiency*, often become organizing principles. When one ordering principle defines a *unique* equilibrium, and an alternative suggests many, the first tends to be applied. *Cultural norms* may also be important.

(4) The properties of the organization matrix are: i. symmetry; ii. null player, iii. linearity in stand alone activities. Only the contribution of each business counts towards overall value added, not the particular number associated with it, (*symmetry*). If a business adds nothing to a coalition, it has zero value, (*null player*). *Linearity* is dealt with in the body of the paper.

- (5) An organization is viable if it adds value, and the whole exceeds the value of stand alone businesses:  $Z(S) \exists 0$  and  $Z(S) \exists \sum_{k \in S} z(b_{kk})$ . Taken together with value creation, viability implies generating positive synergies:  $\sum_{k \in S} z(-^k) \exists 0$
- (6) feasibility implies,  $\sum_{k \in S} Z(\mathbf{a}^k) = Z(S)$ , and value added or rent cannot be merely by forming new coalitions, if  $\sum_{k \in S} Z(\mathbf{a}^k) \exists Z(S)$ , all  $S \in \mathcal{N}$ .
- (7) For a discussion of this and related issues see Matthews (1996).
- (8) The exceptions are the origin and  $g(1) = 0$ .